Mathematica as program support in the integral calculations

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Abstract: In this paper, we give a connection between the mathematical notions and using the computer as educational support at university level. Specifically, mathematical notions used in integral calculations will be explained with help of computer program. The notions, indefinite and definite integral, their calculations and their applications can be easily understand using the computer programs for their presentation. Images obtained with computer programs allows the students to better understand and learn integrals and also give them appropriate training to use this knowledge for current real problems. In this paper we will use mathematical package Mathematica, as computer program to help learning integral calculations.

Keywords: Indefinite integral, definite integral, Mathematica, application of integral calculations

1. INTRODUCTION

In education at university level the most important is the relationship between students and teachers i.e. the way how to transfer information and knowledge from teacher to student. The transfer must be understandable and acceptable from students. This question is the basis of any science, even in mathematics. The development of computer science allows the mathematics teachers to make a connection between computer and mathematics for approaching of mathematical abstractions to the level of understanding among students.

Mathematics as a science especially at university level today cannot be imagined without the use of computer. Some bigger calculations, mathematical concepts and graphic visualizations that previously were treated as an abstraction now with a computer can easily be executed and presented. Also, the calculations which required too much time, as well as 2D graphical visualizations that have been drew manually on the board, can be made with computer in a very short time with much better looking and with wide range of tools for manipulating them. For this we need mathematical packages such as Mathematica [1], [2], [4], [5], [8], MatLab, etc. They are a great tool for education and provide significant improvement in the relationship teacher - student during lectures, as well as for independent work of students and their effective learning of teaching material.

In this paper we will use mathematical package Mathematica, as computer program to help

learning integral calculations in the course Mathematics 2 [2], [6], [7]. The notion definite integrals, its calculation, and its application are a major problem for students and an even greater problem on the teacher how to explain that. Understanding of these notions is much easier using the computer support during teaching hours. Images obtained with computer programs allows the students to better understand and learn integrals and also give them appropriate training to use this knowledge for current real problems.

2. INDEFINITE AND DEFINITE INTEGRAL

Finding a solution $F(x)+C$ of an indefinite integral $\int f(x)\,dx$ should not be a problem for the students, but can be very time consuming. Sometimes, the choice of the method or choice of replacement for solving the integral is a real problem. Therefore, using Mathematica for solving indefinite integral is very simple. For example, the code for solving indefinite integral $\int \dfrac{2x-8}{\sqrt{1-x-x^2}}\,dx$ with more complex integrand function is

$\int \dfrac{2*x-8}{\sqrt{1-x-x^2}}\,dx$ with solution in Mathematica, $2(-\sqrt{1-x-x^2}+\dfrac{9}{2}ArcSin[\dfrac{-1-2x}{\sqrt{5}}])$.

Analogue, the indefinite integral $\int \dfrac{x^7+x^3}{x^{12}-2x^4+1}\,dx$ have code $\int \dfrac{x^7+x^3}{x^{12}-2*x^4+1}\,dx$ with solution in Mathematica

$$\dfrac{1}{20}(-(5+2\sqrt{5})Log[-1+\sqrt{5}-2x^4]+10\,Log[-1+x^4]+(-5+2\sqrt{5})Log[-1+\sqrt{5}-2x^4])$$

where $Log\,[x]$ is $\ln x$. To note that Mathematica in the solution of indefinite integral omitted the constant C which is usually added to mark the set of functions as solution of indefinite integral.

For given not negative integral function $f(x)$ defined on a closed interval $[a,b]$, a definite integral $\displaystyle\int_a^b f(x)\,dx$ is defined as surface area of the revolution in xy - plane bounded by the graph of the function $f(x)$, the vertical lines $x=a, x=b$ and the x - axis. For example, the definite integral $\displaystyle\int_0^8 (\sqrt{2x}+\sqrt[3]{x})\,dx$ with code $\displaystyle\int_0^8 (\sqrt{2*x}+\sqrt[3]{x})\,dx$ has the solution $\dfrac{100}{3}$ in Mathematica. The geometrical interpretation is: the surface area of the revolution in xy - plane bounded by the not negative integral function $f(x)=\sqrt{2x}+\sqrt[3]{x}$, the vertical lines $x=a, x=b$ and x-axis is $\dfrac{100}{3}$ square units, is shown in figure 1,

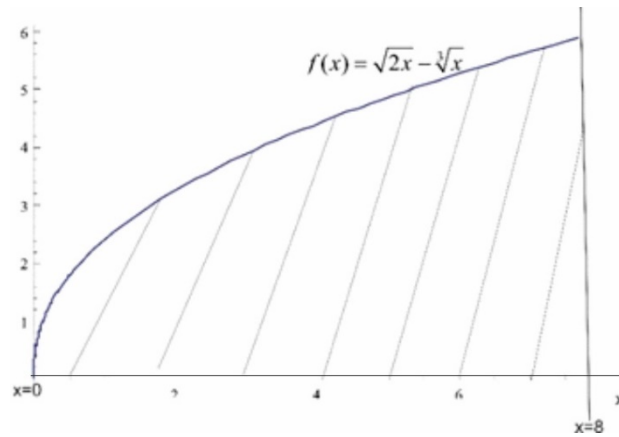**Figure 1.** *The region for the definite integral* $\int\limits_{0}^{8}(\sqrt{2x}+\sqrt[3]{x})\,dx$

If the solution of definite integral is not integer, finite decimal number or cannot be represented as fractions, then we are changing the code in Mathematica. For example, for the

definite integral $\int\limits_{0}^{\frac{\pi}{4}}x\sqrt{tg\,x}\,dx$ with code $\int\limits_{0}^{\frac{\pi}{4}}x*Sqrt\,[Tan\,[x]]\,dx\,//\,N$ we have the solution

0.233181 which is surface area of the revolution in the xy-plane bounded by the graph of the

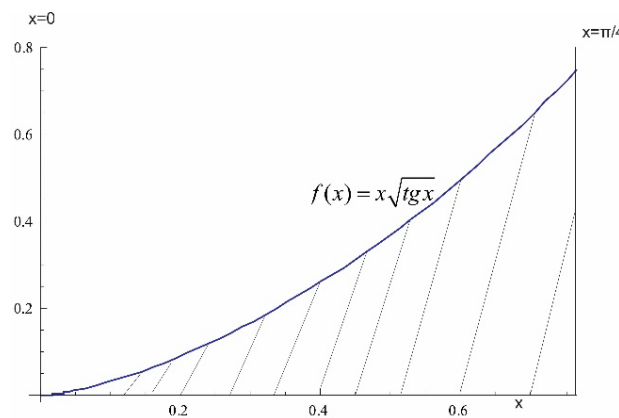function $f(x)=x\sqrt{tg\,x}$, the vertical lines $x=0, x=\dfrac{\pi}{4}$ and x - axis is shown in figure 2.

Figure 2. *The region for the definite integral* $\int\limits_{0}^{\frac{\pi}{4}}x\sqrt{tg\,x}\,dx$

3. APPLICATIONS OF INTEGRAL CALCULATIONS IN GEOMETRY

Applications of integral calculations for calculate the surface area of planar figure that is bounded by the graphs of $f(x), g(x)\, (f(x) \geq g(x), x \in [a,b])$ and the vertical lines $x = a, x = b$ is given with the definite integral $\int_a^b (f(x) - g(x))\, dx$. For example, the surface area of planar figure that is bounded by the graphs of $f(x) = 2 - x^2, g^3(x) = x^2$ can be calculated in three steps as follows:

- We define the bounds of definite integral by finding the intersection of the curves $f(x) = 2 - x^2, g^3(x) = x^2$ with the code $Solve[y == 2 - x^2, y^3 == x^2, \{x, y\}]$ where we take into account only the real solutions $(-1,-1), (1,1)$;

- We draw the graphs of the functions $f(x) = 2 - x^2, g(x) = x^{\frac{2}{3}}$ as in figure 3 in order to find the area of integration. The function $f(x) = 2 - x^2$ is marked with blue color and the function $g(x) = x^{\frac{2}{3}}$ with red color.

- We calculate the definite integral $\int_{-1}^{1} (2 - x^2 - x^{\frac{2}{3}})\, dx$ with code $\int_{-1}^{1} (2 - x^2 - \sqrt[3]{x^2})\, dx$ which gives solution $\frac{32}{15}$ square units.
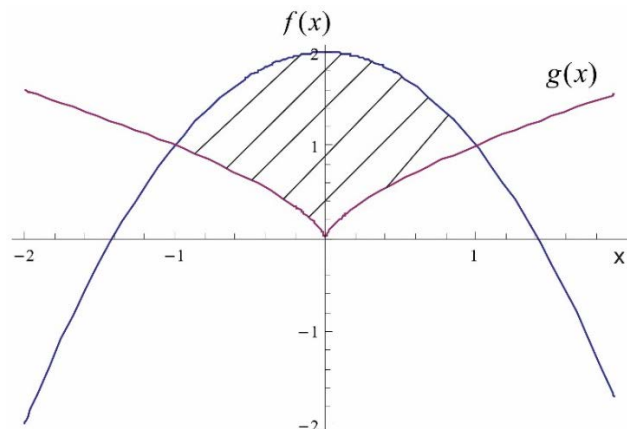

Figure 3. *The graphs of the functions* $f(x) = 2 - x^2, g(x) = x^{\frac{2}{3}}$

The integral calculation has application for calculating the arc length of curve $y = f(x)$ between two points in xy-plane with first coordinates $x = a, x = b, a < b$ and it is calculated

with a definite integral $\int_a^b \sqrt{1 + f'^2(x)}\, dx$. For presenting, we will use the curve with equation

$y = \ln x$. For this curve we will calculate the arc length in the interval of $\sqrt{3}$ to $\sqrt{8}$ by the

formula $\int_{\sqrt{3}}^{\sqrt{8}} \sqrt{1 + (\frac{1}{x})^2}\, dx$ for $y' = \frac{1}{x}$. The code in Mathematica is $\int_{\sqrt{3}}^{\sqrt{8}} \sqrt{1 + \frac{1}{x^2}}\, dx$, where

$\partial_x \, Log = \frac{1}{8}$ give the solution $\frac{1}{2}(2 + Log\,[\frac{3}{2}])$ for $Log\,[\frac{3}{2}]$ or $\ln \frac{3}{2}$.

Use of Mathematica in integral calculations for calculate a volume of rotary body and area of rotary body layer, gives more precise graphic visualization of the objects in 3D space.

The task to find the volume of the rotary body obtained with rotating of the graph of the curve $y = f(x)$ around x - axis on the interval $[a, b]$ is solved with the definite integrals $\pi \int_a^b f^2(x)\, dx$ and its surface area is obtained by definite integral $2\pi \int_a^b f(x) \sqrt{1 + f'^2(x)}\, dx$, respectively.

For example if we want to find the volume of the rotary body and its surface area obtained with rotating of the graph of the curve around x - axis is the semicircle with the equation $y = \sqrt{a^2 - x^2}$, where $a > 0$ is its radius. In figure 4 a) is shown the graph of function $y = \sqrt{5^2 - x^2}$ and in figure 4 b) is shown the rotation around x – axis where rotary body is the ball.

Be brief and give most important conclusion from your paper. Do not use equations and figures here.

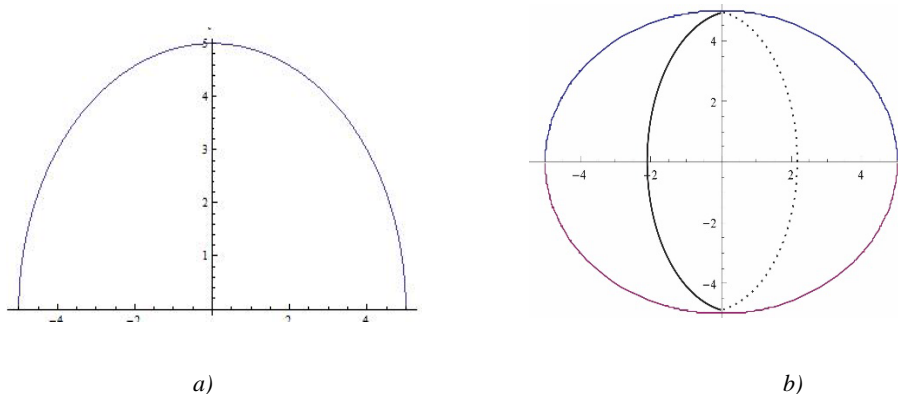a)   b)

Figure 4. a) The graph for the curve $y = \sqrt{5^2 - x^2}$ b) rotating body - the ball

By using integral calculation the volume of the ball is $\pi \int\limits_{-a}^{a} (\sqrt{a^2 - x^2})^2 \, dx$ with code

$\pi \int\limits_{-a}^{a} (\sqrt{a^2 - x^2})^2 \, dx$ in Mathematica and has solution $\dfrac{4a^3 \pi}{3}$ square units. The surface area

is calculated with the definite integral $2\pi \int\limits_{-a}^{a} \sqrt{a^2 - x^2} \sqrt{1 + (-\dfrac{x}{\sqrt{a^2 - x^2}})^2} \, dx$ with code

$2\pi \int\limits_{-a}^{a} \sqrt{a^2 - x^2} * \sqrt{1 + (-\dfrac{x}{\sqrt{a^2 - x^2}})^2} \, dx$ in Mathematica where the derivate of the

function $y = \sqrt{a^2 - x^2}$ with code $\partial_x \sqrt{a^2 - x^2}$ is $-\dfrac{x}{\sqrt{a^2 - x^2}}$. The solution of the

definite integral is $4a\pi Abs[a]$ where for $a > 0$ is obtained $Abs[a] = |a| = a$. Finally, the

solution of the definite integral is $4a\pi Abs[a] = 4\pi a^2$ that is solution obtained with the basic

geometry for surface area of sphere with radius a .

The task is more complex for rotating body obtained with rotary of the plane figure boundary with two curves $f(x), g(x) (f(x) \geq g(x), x \in [a,b))$ and the vertical lines $x = a, x = b$ around x – axis where the volume and its surface area are calculated with the definite integrals

$\pi \int\limits_{a}^{b} [f^2(x) - g^2(x)] \, dx$ and $2\pi \int\limits_{a}^{b} [f(x)\sqrt{1 + f'^2(x)} - g(x)\sqrt{1 + g'^2(x)}] \, dx$, respectively.

For example, the volume of rotary body obtained with rotation of the circle $x^2 + (y - b)^2 = a^2, (b \geq a > 0)$ around x – axis, can be calculated in two steps:

1. From the equation of circle $x^2 + (y - b)^2 = a^2$ with radius a and center in the point $(0,b)$ are obtained two different functions $y = b + \sqrt{a^2 - x^2}, y = b - \sqrt{a^2 - x^2}$. In figure 5, is shown the circle $x^2 + (y - 5)^2 = 2^2$ where the function $y = 5 - \sqrt{2^2 - x^2}$ is marked with red color and the function $y = 5 + \sqrt{2^2 - x^2}$ with blue color.
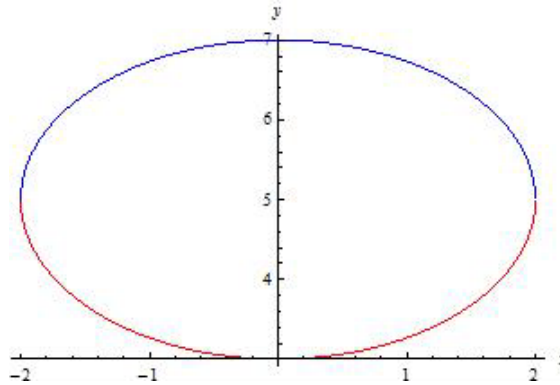
Figure 5: *The graphs of the functions* $y = 5 - \sqrt{2^2 - x^2}$ *and* $y = 5 + \sqrt{2^2 - x^2}$

2. By rotation of this area, we obtain the rotary body where the volume is calculated with the definite integral $\pi \int_{-a}^{a} [(b + \sqrt{a^2 - x^2})^2 - (b - \sqrt{a^2 - x^2})^2] \, dx$ the code in Mathematica is $\pi * \int_{-a}^{a} [(b + \sqrt{a^2 - x^2})^2 - (b - \sqrt{a^2 - x^2})^2] \, dx$. This definite integral has the solution $2a\sqrt{a^2}\, b\pi^2$ and for $a > 0$, the finally obtained solution is $2\pi^2 a^2 b$.

To calculate the surface area of rotary body, we use the definite integral

$$2\pi \int_{-a}^{a} [(b + \sqrt{a^2 - x^2})\sqrt{1 + (-\frac{x}{\sqrt{a^2 - x^2}})^2} - (b - \sqrt{a^2 - x^2})\sqrt{1 + (\frac{x}{\sqrt{a^2 - x^2}})^2}] \, dx$$

with code

$$2 * \pi * \int_{-a}^{a} [(b + \sqrt{a^2 - x^2}) * \sqrt{1 + (-\frac{x}{\sqrt{a^2 - x^2}})^2} - (b - \sqrt{a^2 - x^2}) * \sqrt{1 + (\frac{x}{\sqrt{a^2 - x^2}})^2}] \, dx$$

and obtained solution $8a\,\pi Abs[a]$ where for $a > 0$, $Abs[a] = |a| = a$, therefore the solution is $8a\,\pi\,Abs[a] = 8\pi a^2$. The first derivation for the functions $y = b + \sqrt{a^2 - x^2}$ and $y = b - \sqrt{a^2 - x^2}$ can be obtained by the codes $\partial_x (b + \sqrt{a^2 - x^2})$ and $\partial_x (b - \sqrt{a^2 - x^2})$, respectively in Mathematica.

The same steps used for x axes can be used for calculation of volume and surface area of rotary body obtained with rotation of planar curve around y-axis. In this case integral

calculation will be made for variable y and sub integral functions will be $x = f(y), x = g(y)$

4. CONCLUSION

In this paper we give brief review of using Mathematica in educational purpose. Computer program Mathematica can be used by teachers as a tool for easily explaining integral calculation to students and also can be used by students to easily understanding these topics. This can help to improve student knowledge about integrals and its application in real problems.

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